4. STRUCTURAL DESIGN

General. The following structural layout and design criteria and methods should not be used when F + h is greater than 20 ft or when F is greater than 15 ft. Where these limits are exceeded, a more conservative, complete, and careful analysis is required.

Proportions Required for Stability. The proportions, other than those determined hydraulically, are designed to provide a stable structure. These proportions are often mathematically indeterminate and must be based partially on the designer's experience plus careful consideration of possible mode of failure. Sliding, piping, uplift, undermining, fill slopes, and lateral scour must be considered and analyzed as accurately as possible.

The purposes of the various structural parts are as follows: The headwall extension is to permit a stable fill and to prevent piping around the structure. The cutoff wall is to prevent piping under the structure, to reduce uplift pressures, and to resist sliding. The toewall is to prevent piping under the structure and to prevent undermining of the apron. The sidewall is to hold a stable fill and protect it against erosion due to water passing over the spillway. The wingwall is to hold a stable fill and to prevent serious scour of the fill and gully banks.

The design problem varies greatly with site conditions. In locations where the ground water elevation is a considerable distance below the foundation, the foundation is permeable, and the fill around the structure is normally dry, the problems of piping and uplift are insignificant and the other dangers are greatly reduced.

Horizontal Pressures. Horizontal earth pressures are affected by numerous factors, such as the characteristics of the backfill material against the wall, the relative permeability of the foundation material and the backfill material, the elevation of the water table, and the backfill drainage provided.

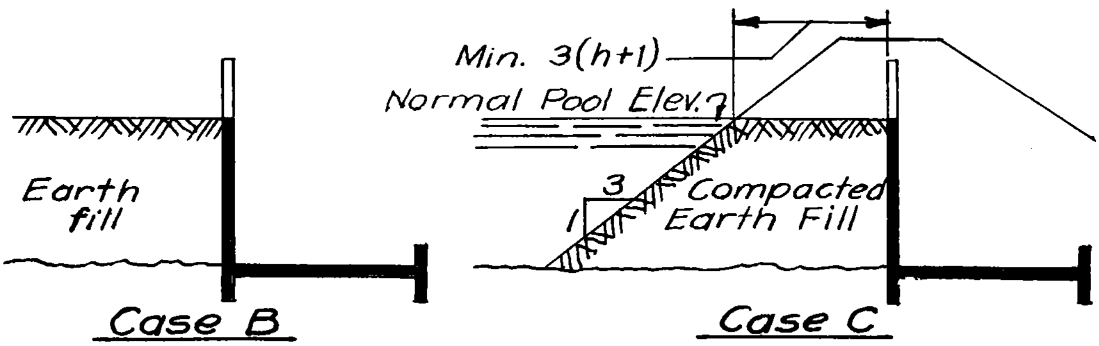
The soil characteristics that affect the horizontal earth pressures are permeability, cohesion, angle of internal friction, weight, void ratio, and moisture content. Refer to Engineering Handbook, Section 6 on Structural Design, part 2.2.2.

Loads on Headwall. The method discussed in the following paragraphs for the determination of the loads on a headwall of a drop spillway is based on judgment and past experience. It is believed that this method results in safe design values and may be applied in the design of drop spillways where F is 15 feet or less and F + h is 20 feet or less. In discussing this procedure we will first list and define the variables that affect the equivalent fluid pressures and then cite a numerical example to clarify its use. The following different conditions of backfill must be considered.

Case A. No fill against headwall; therefore, the pressure against the headwall is equal to full hydrostatic pressure.

Case B. Gully graded full to crest elevation.

Case C. An earth fill berm constructed to crest elevation.



Relative Permeability of the Foundation and the Backfill. The following 3 conditions of relative permeability will be considered: the permeability of the foundation is greater than, equal to, and less than the permeability of the backfill.

Effect of Water Table Elevation. The elevation of the water table above and below the spillway, before and after construction, has a significant effect on loads on the headwall and other elements of the design.

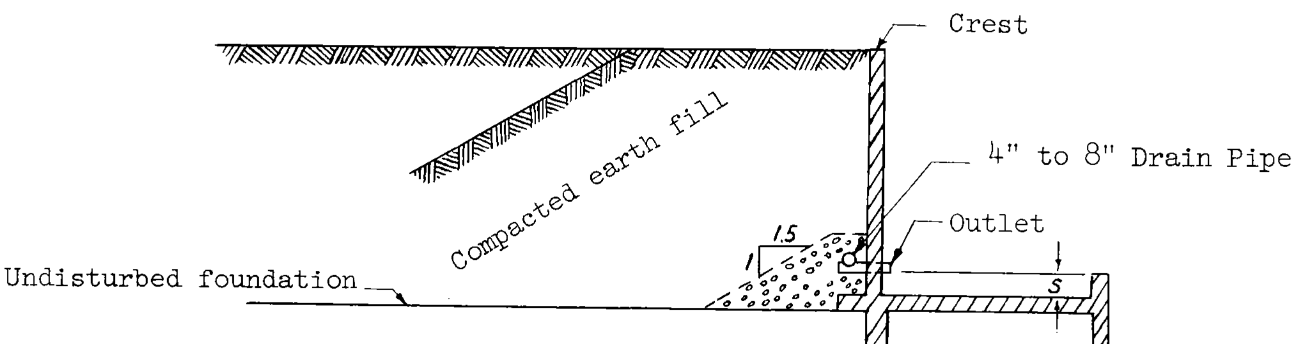
If the water table is low and the foundation material is relatively homogeneous and permeable, the flow of water from a reservoir or from percolation through backfill in the channel above the dam tends to pass downward through the foundation in a more or less vertical direction, until it merges with the subsurface flow. The increase in the discharge of subsurface flow will result in a rise in the ground-water elevation at the site; the amount of rise will depend upon the permeability of the foundation, the increase in ground-water discharge, and other factors. In such a case there will be no increase in horizontal pressure on the headwall, due to saturation of an earth backfill, under these conditions: (1) the rise in ground-water elevation does not create hydrostatic pore pressures on the base of the spillway, and (2) either the backfill is homogeneous and more nearly impervious than the foundation, or there is a continuous increase in permeability along the flow lines of the percolating waters.

However, if the water table is high (i.e., close to or above proposed apron elevation) prior to construction, or would be raised to such an elevation by works of improvement downstream, quite a different situation prevails. In such a case, a differential head created by the dam will result in uplift pressures on the base of the spillway and increased pressure on the headwall. The magnitude of this uplift and increased headwall pressure will depend upon the total differential head, type and efficiency of drainage provided above the headwall, relative permeability of the backfill above the headwall and various strata in the foundation, depth of cutoff and toewalls, physical characteristics of backfill and foundation soils, tailwater elevation, and perhaps other factors. With uplift is associated the possibility that the escape gradient of pore pressure below the spillway will be sufficient to cause piping.

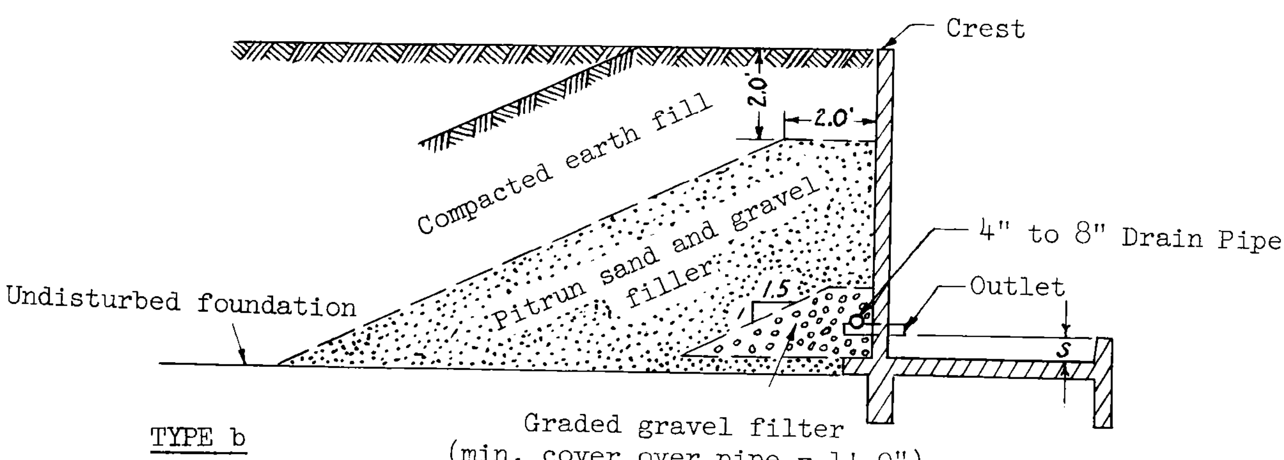
Uplift and increased pressures on the headwall are apt to occur, even though the true water table is well below the foundation of the spillway, if a continuous layer of impervious or relatively impervious material exists in the foundation near the surface and this layer or strata is covered with permeable material. Increased pressures on the headwall are certain, and uplift pressures in excess of tailwater will occur unless all flow underneath the spillway is prevented by a watertight cutoff wall which extends well into the impervious strata. This situation is comparable in general to the situation created by a high ground-water elevation, and should be considered so in the estimation of headwall loads.

There are many shades of gray between the picture of black or white presented above. However, thorough studies in soil mechanics to define the flow net with reasonable accuracy are seldom justified in the design of average-size drop spillways. If there is reasonable doubt about the existence of a low-water-table condition, then a high-water-table condition should be assumed for the design.

<u>Drainage of Fill Against Headwall</u>. Two types of drains will be considered. Perforated pipe or porous concrete pipe will be used in both types and should extend a distance equal to F beyond the edges of the weir opening. The difference in the two types will be in the size and design of the gravel filters.



<u>TYPE a</u>



<u>TYPE b</u>

<u>FIGURE 4.1</u>

The criteria for the design of the gradation of the filters are discussed in the drop spillway design example.

The selection of the type of drain will be governed by economics and stability design of the structure. It may be necessary to use the type B drain to insure stability against sliding and piping. In locations where filter materials are readily available at a conservative cost, it is recommended that the type B drain always be considered.

Table 4.1 (page 4.5) furnishes a method of estimating the elevation of the saturation line (y_2) above the top of the apron for all combinations of the variables for no-flow and design-discharge-flow conditions. When the table shows that y_2 is greater than y_o, the backfill will be considered saturated to crest elevation. Such a table represents an obvious over simplification of the problem. However, reasonable care in the interpretation of foundation soil borings and conservative use of the table should give results that are practical and within permissible limits of error. Please note drainage requirements listed in table 4.1 (page 4.5).

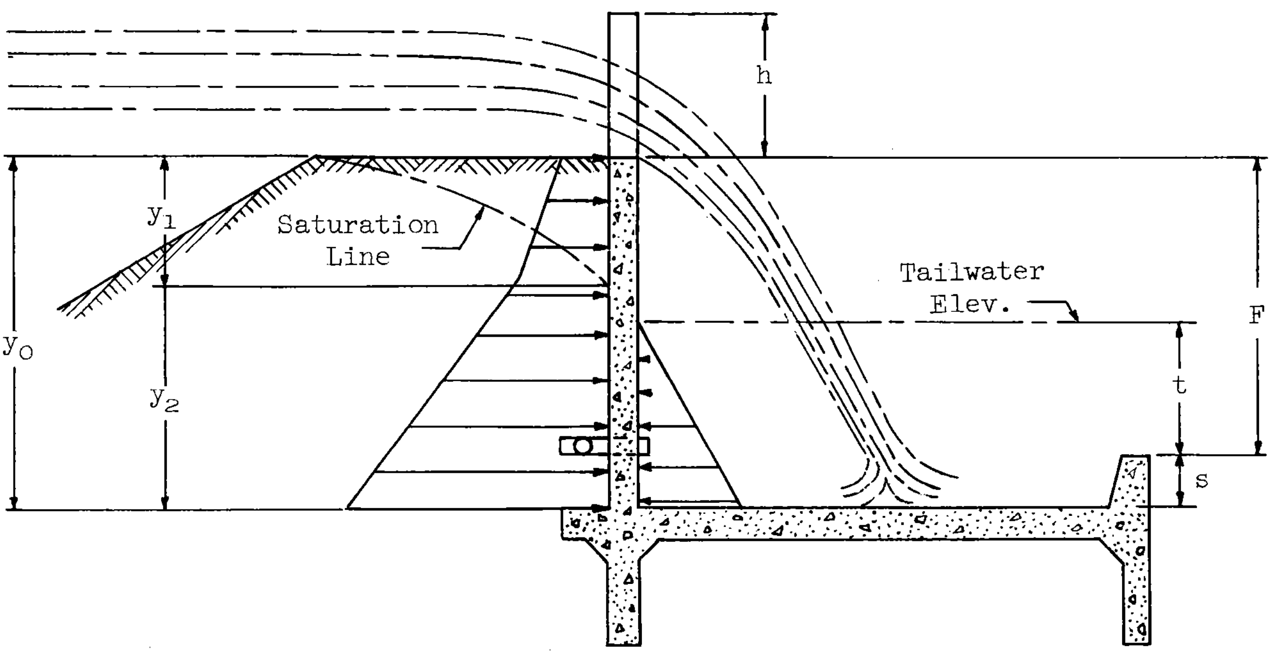


FIGURE 4.2

The headwall is designed as a slab fixed at the bottom and two vertical edges and free at the top. In order to use the moment and shear coefficients of ES-6, Engineering Handbook, Section 6 on Structural Design, the actual load diagram must be resolved into a triangular load diagram. This is done by equating the cantilever moments at the base of the headwall and solving for w, the equivalent fluid pressure producing the triangular load diagram.

This procedure, along with the use of table 4.1 (page 4.5), can best be explained by examples.

CASE	WATER TABLE	RELATIVE PERMEABILITY OF FOUNDATION TO BACKFILL	DRAINAGE	(See fig. 4.2, page 4.4) y_2 = NO FLOW	FULL FLOW	PIPING A PROBLEM
A	High	---	None	y_o	y_o	Yes
	Low	---	None	y_o	y_o	No
B	High	greater	a	s + 0.3F	t + s + 0.3F	Yes
		greater	b	s + 0.1F	t + s + 0.1F	Yes
		equal	a	s + 0.4F	t + s + 0.4F	Yes
		equal	b	s + 0.15F	t + s + 0.15F	Yes
		less	a	s + 0.5F	t + s + 0.5F	Yes
		less	b	s + 0.2F	t + s + 0.2F	Yes
B	Low	greater	None	0	0	No
		equal	None	0	0	No
		less	a	s + 0.3F	t + s + 0.3F	No
		less	b	s + 0.1F	t + s + 0.1F	No
C	High	greater	a	s + 0.4F	t + s + 0.4F	Yes
		greater	b	s + 0.1F	t + s + 0.1F	Yes
		equal	a	s + 0.5F	t + s + 0.5F	Yes
		equal	b	s + 0.15F	t + s + 0.15F	Yes
		less	a	s + 0.6F	t + s + 0.6F	Yes
		less	b	s + 0.2F	t + s + 0.2F	Yes
C	Low	greater	None	0	0	No
		equal	None	0	0	No
		less	a	s + 0.3F	t + s + 0.3F	No
		less	b	s + 0.1F	t + s + 0.1F	No

TABLE 4.1

Example 4.1

Given: Drop spillway with F = 8.0 ft, h = 3.0 ft, d_c = 1.80 ft, s = 1.0 ft, t = 2.5 ft, H = 2.50 ft

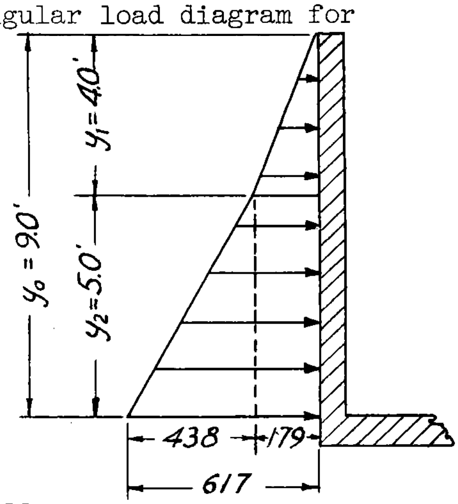
Relative permability; foundation = backfill

Case C (page 4.2)

Backfill properties

	Earth	Pitrun sand and gravel
dry wt. lbs/ft^3	100	118
e = void ratio	0.65	0.45
percent voids	39.4	31.0
moist wt. lbs/ft^3	110	125
ϕ	25^0	35^0
cohesion	0	0
eff. subm. wt. lbs/ft^3	62	65

Find: w, equivalent fluid pressure of triangular load diagram for
(1) No flow, type (a) drainage
(2) No flow, type (b) drainage
(3) With flow, type (a) drainage
(4) With flow, type (b) drainage



Solutions: (1) No flow, type (a) drainage: For type (a) drainage consider the backfill as earth for the total height of the headwall. From table 4.1 (page 4.5)

y_2 = s + 0.5F = 1.0 + 4.0 = 5.0 ft

p_a = unit active lateral earth pressure, psf

$$p_a = W\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right)$$

where ϕ = angle of internal friction of backfill

W = vertical weight of material lbs/ft^2
= vertical pressure

$\dfrac{1 - \sin \phi}{1 + \sin \phi}$ = ratio of lateral pressure to vertical pressure

At crest elevation, $p_a = 0$

At 4.0 ft below crest

vert wt = 4 · 110 = 440 lbs

$$p_a = 440\left(\frac{1 - \sin 25^0}{1 + \sin 25^0}\right) = 440 \cdot 0.406 = 179 \text{ lbs/ft}^2$$

At 9.0 ft below crest

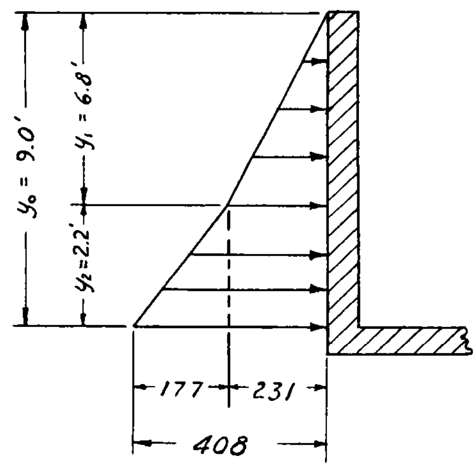
$$\text{vert intergranular pressure} = 440 + (5 \cdot 62) = 750 \text{ lbs/ft}^2$$

$$p_a = (750 \cdot 0.406) + (62.4 \cdot 5) = 617 \text{ lbs/ft}^2$$

$$\frac{wy_o^3}{6} = (179 \cdot 2.0 \cdot 6.33) + (179 \cdot 5.0 \cdot 2.5) + (438 \cdot 2.5 \cdot 1.67)$$

$$w = 6\,\frac{(2270 + 2240 + 1830)}{729} = 52.2 \text{ lbs/ft}^3 = \text{unit weight of equivalent fluid}$$

(2) No flow, type (b) drainage:
For type (b) drainage consider the backfill as pit-run sand and gravel for the total height of the headwall. From table 4.1 (page 4.5)



$$y_2 = s + 0.15F = 1.0 + 1.2 = 2.2 \text{ ft}$$

$$p_a = W\left(\frac{1 - \sin 35^o}{1 + \sin 35^o}\right) = 0.272\ W$$

At crest elevation, $p_a = 0$

At 6.8 ft below crest

$$W = 125 \cdot 6.8 = 850 \text{ lbs/ft}^2$$

$$p_a = 850 \cdot 0.272 = 231 \text{ lbs/ft}^2$$

At 9.0 ft below crest

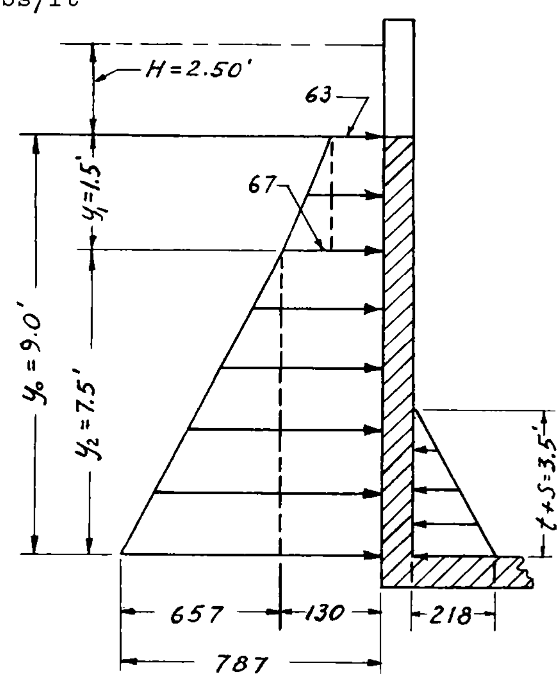
$$W = 850 + (2.2 \cdot 65) = 993 \text{ lbs/ft}^2$$

$$p_a = (993 \cdot 0.272) + (62.4 \cdot 2.2) = 270 + 138 = 408 \text{ lbs/ft}^2$$

$$w = \frac{6}{729}\left[(231 \cdot 3.4 \cdot 4.47) + (231 \cdot 2.2 \cdot 1.1) + (177 \cdot 1.1 \cdot 0.733)\right]$$

$$w = \frac{6}{729}(3510 + 560 + 143) = 34.7 \text{ lbs/ft}^3$$

(3) With flow, type (a) drainage
From fig. 5.1 (page 5.2)



$$t = 2.5 \text{ ft for } k = 1.15 \text{ ft}$$

$$t + s = 2.5 + 1.0 = 3.5 \text{ ft}$$

$$y_2 = t + s + 0.5F$$

$$y_2 = 3.5 + 4.0 = 7.5 \text{ ft}$$

At crest

$$W = 62.4 \cdot 2.5 = 156 \text{ lbs/ft}^2$$

$$p = 156 \cdot 0.406 = 63 \text{ lbs/ft}^2$$

At 1.5 ft below crest

$$W = 156 + (110 \cdot 1.5) = 321 \text{ lbs/ft}^2$$

$$p = 321 \cdot 0.406 = 130 \text{ lbs/ft}^2$$

At 9.0 ft below crest

$$W = 321 + (7.5 \cdot 62) = 786 \text{ lbs/ft}^2$$

$$p = (786 \cdot 0.406) + (7.5 \cdot 62.4) = 319 + 468 = 787 \text{ lbs/ft}^2$$

Tailwater pressure at apron elevation

$$p = 3.5 \cdot 62.4 = 218 \text{ lbs/ft}^2$$

$$w = \frac{6}{729}\Big[(63 \cdot 1.5 \cdot 8.25) + (67 \cdot 0.75 \cdot 8.0) + (130 \cdot 7.5 \cdot 3.75) + (657 \cdot 3.75 \cdot 2.5) - (218 \cdot 1.75 \cdot 1.17)\Big]$$

$$w = \frac{6}{729}(780 + 402 + 3660 + 6160 - 445)$$

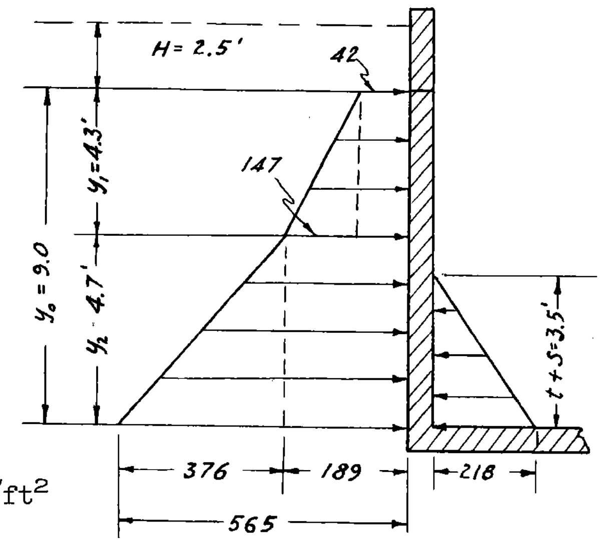
$$w = \frac{6}{729}(10{,}557) = 86.9 \text{ lbs/ft}^3$$

(4) With flow, type (b) drainage

$$t + s = 3.5$$

$$y_2 = t + s + 0.15F$$

$$y_2 = 3.5 + 1.2 = 4.7$$



At crest

$$W = 62.4 \cdot 2.5 = 156 \text{ lbs/ft}^2$$

$$p = 156 \cdot 0.272 = 42 \text{ lbs/ft}^2$$

At 4.3 ft below crest

$$W = 156 + (4.3 \cdot 125) = 693 \text{ lbs/ft}^2$$

$$p = 693 \cdot 0.272 = 189 \text{ lbs/ft}^2$$

At 9.0 ft below crest

$$W = 693 + (4.7 \cdot 65) = 998 \text{ lbs/ft}^2$$

$$p = (998 \cdot 0.272) + (62.4 \cdot 4.7) = 565 \text{ lbs/ft}^2$$

Tailwater pressure

$$p = 3.5 \cdot 62.4 = 218 \text{ lbs/ft}^2$$

$$w = \frac{6}{729}\Big[(42 \cdot 4.3 \cdot 6.85) + (147 \cdot 2.15 \cdot 6.13) + (189 \cdot 4.7 \cdot 2.35) + (376 \cdot 2.35 \cdot 1.57) - (218 \cdot 1.75 \cdot 1.17)\Big]$$

$$w = \frac{6}{729}(1237 + 1935 + 2087 + 1387 - 445)$$

$$w = \frac{6}{729}(6201) = 51.1 \text{ lbs/ft}^3$$

Loads on Sidewalls and Wingwalls. For relatively low walls, the equivalent fluid pressures shown in table 6.2-1, Engineering Handbook, Section 6 on Structural Design, may be used as a guide. In the design of large structures, which justify more careful investigations, it is recommended that the graphical method explained in paragraph 2.2.2 of the Structural Design Section be employed to determine the equivalent fluid pressure.

Loads on Headwall Extensions. When the headwall extension is designed monolithically with the rest of the structure, there is a possibility of a differential pressure in the downstream direction or in either direction at different elevations of the wall. If the structure is stable against sliding, without the passive resistance of the earth on the downstream side of the headwall extension coming into play, the differential pressure acting on the wall will be the difference in active earth pressures on both sides of the wall. If this passive resistance is required to stabilize the structure against sliding, the differential pressure will be the difference of the active pressure on the upstream side and the passive pressure on the downstream side. These differential pressures are highly indeterminate. It is, therefore, recommended that the headwall extension be designed for a differential equivalent fluid pressure of 5 to 10 pounds per cubic foot, with the assumption that it may occur in either direction.

For high headwall extension, or where the possibility of differential settlement makes the designer doubtful about using the above assumption, it is suggested that the headwall extension be made articulate from the rest of the structure. The headwall extension will then act as a diaphragm and need be reinforced only to meet the minimum steel requirements. The joint between the headwall extension and the rest of the structure must be made water tight by the use of a continuous rubber water stop or some other equally suitable device.

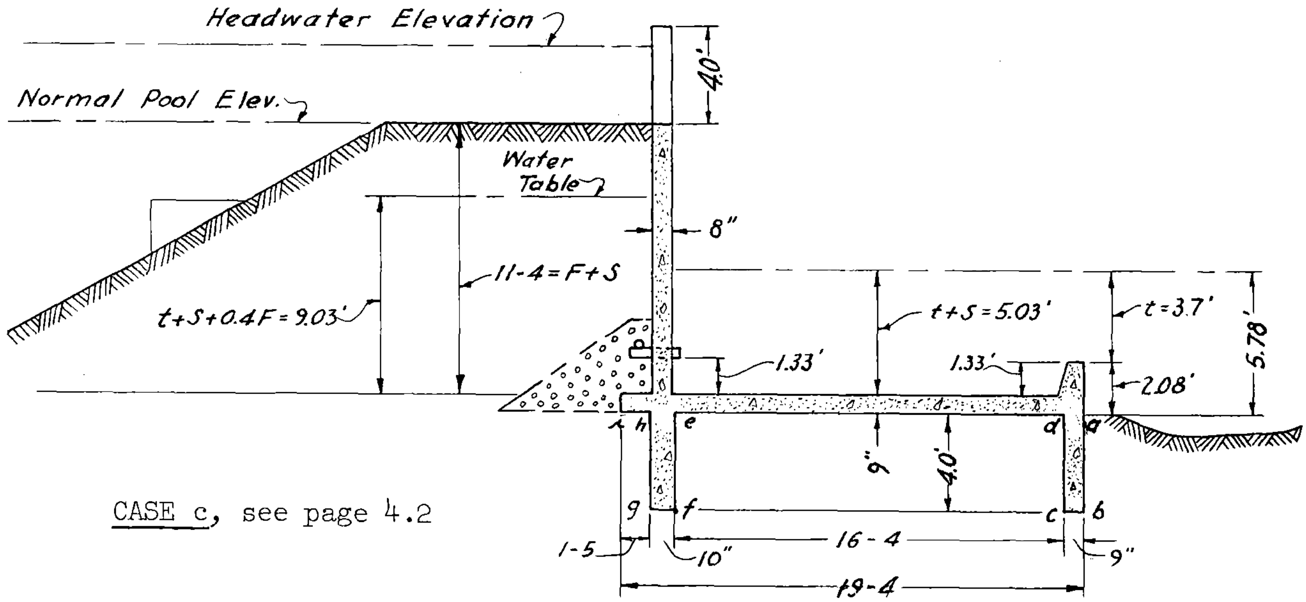
Uplift. Upward hydrostatic pressures may exist on the base of the spillway, as the result of pressure transmitted through the water in a saturated foundation material. If a differential in head exists between the elevation of the water surfaces above and below the spillway, flow or movement of the water will take place and the uplift pressures will vary with the pressure gradient.

For earth foundations, these uplift pressures are assumed to exist over the entire base area of the spillway.

Uplift pressure can be roughly estimated by the "line of creep" theory, see "Piping" page 4.14. The procedure is explained by the following example.

Example 4.2

Given: Drop spillway, F = 10.0 ft, h = 4.0 ft, d_c = 2.67 ft, s = 1.33 ft, and t = 3.7 ft (See sketch, page 4.10). Relative permeability of foundation material is greater than fill material. Type (a) drainage used above headwall. (See page 4.3)



CASE c, see page 4.2

Find: Uplift pressures on base of structure and draw uplift diagram for with-flow condition.

Solution: Step 1. Find hydrostatic pressure at point a. Assume downstream channel has eroded to elevation of bottom of apron.

Required depth of tailwater above top of transverse sill = 3.7 ft for d_c = 2.67 ft, see fig. 5.1 (page 5.2) with k = 1.0.

Depth of tailwater above point a = 3.70 + 1.33 + 0.75 = 5.78 ft.

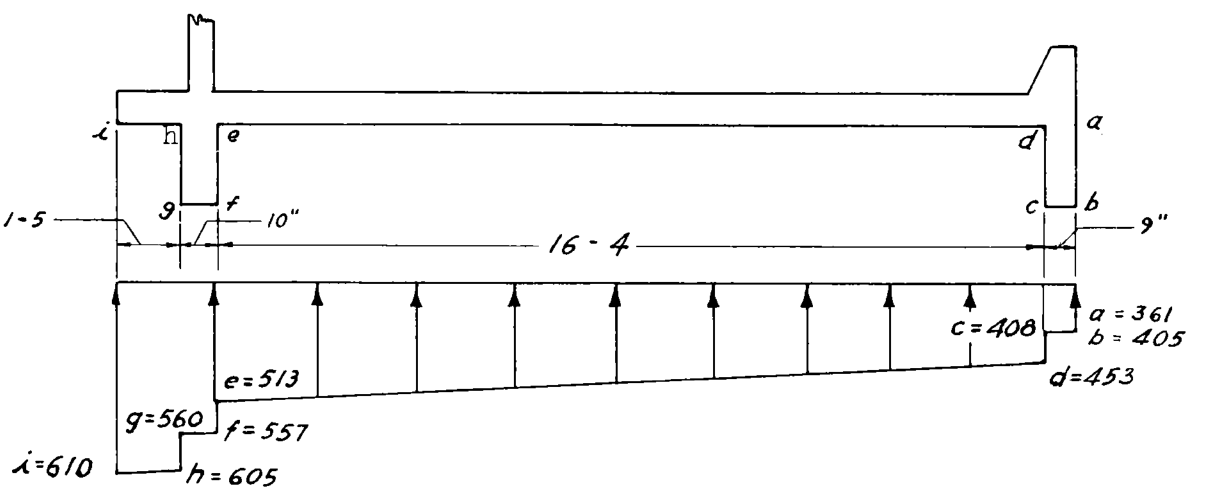
Hydrostatic pressure at point a = 5.78 · 62.4 = 361 lbs/ft^2

Step 2. Find hydrostatic pressure at point i. Estimated elevation of water table above top of apron = t + s + 0.4F, table 4.1 (page 4.5). t + s + 0.4F = 5.03 + 4.0 = 9.03 ft. Elevation of water table above bottom of apron = 9.03 + 0.75 = 9.78 ft. Hydrostatic pressure at point i = 9.78 · 62.4 = 610 lbs/ft^2.

Step 3. Compute the total weighted creep distance and the change in pressure per foot of weighted creep distance. It is assumed that the pressures vary between points a and i in direct proportion to the weighted creep distance. The weighted creep distance = (bc + de + fg + hi) ÷ 3 + ab + cd + ef + gh = (19.33 ÷ 3) + (4 · 4) = 22.44 ft. The change in pressure per foot of weighted creep distance = (610 − 361) ÷ 22.44 = 11.07 lbs/ft^2.

Step 4. Calculate the pressures at various points and obtain the total uplift on a one foot slice as illustrated by the following tabulation.

Point	Weighted creep distance between points ft	Increase in pressure between points psf	Pressure at point psf	Average pressure between points psf	Base area between points ft^2	Uplift between points lbs
a			361			
	4.0	44.4				
b			405			
	0.25	2.8		406.5	0.75	305
c			408			
	4.0	44.4				
d			453			
	5.44	60.3		483.0	16.33	7,887
e			513			
	4.0	44.4				
f			557			
	0.28	3.1		558.5	0.83	464
g			560			
	4.0	44.4				
h			605			
	0.47	5.2		607.5	1.42	863
i			610			
TOTALS	22.44	249.0			19.33	9,519



The sum of the last column in the tabulation gives the total uplift per foot width of the structure as 9,519 lbs. The uplift diagram shows the unit uplift at any point along the base.

For adequate computation of loads on the spillway apron, it is also necessary to compute the uplift pressures for the no-discharge condition. For this condition the pressure at point a is zero. At point i the pressure would be $62.4\ (s + 0.4F + 0.75) = 62.4\ (1.33 + 4.0 + 0.75) = 380$ lb/ft^2, see table 4.1 (page 4.5).

<u>Contact Pressures</u>. The total upward load on the base of the spillway can be divided into 2 parts: (1) the uplift described above, and (2) contact pressures. The contact pressures are transmitted into the foundation by direct contact of the foundation material with the spillway. Obviously, the total upward load on the base of the spillway, which consists of both uplift and contact pressures, must equal the sum of all weights and other downward forces.

The distribution of the contact pressures over the base of the structure depends upon the rigidity of the structure, the characteristics of the foundation material, and the magnitude of the resultant overturning moment acting on the structure. This pressure distribution is highly indeterminate.

It is common engineering practice to assume that the vertical foundation contact pressures vary in a straight-line relationship along any longitudinal section parallel to the center line of the spillway, and that these pressures are constant along any section taken at right angles to the center line. The following procedure for computing these contact pressures agrees with these assumptions.

Contact pressures should be computed for the following loading conditions:

1. Before any backfill has been placed around the spillway.
2. After all backfill has been placed, but without flow over the spillway.
3. With the spillway operating at design discharge capacity.

Good design requires that the contact pressures be compressive in nature over the entire base of the structure. Should an analysis, made in accordance with the following procedure, indicate contact pressures that tend to separate the structure from its foundation at any point (tension), the proportions of the spillway must be changed sufficiently to overcome this condition.

Headwall extensions and wingwalls should be ignored in computing contact pressures; in long weirs it is permissible to deal with a typical bay or longitudinal segment of the spillway. In either case, the area over which the contact pressures are assumed to exist will be a rectangle. It is assumed that the spillway is symmetrical about a longitudinal center line parallel to the direction of flow.

The equation for computing contact pressures for a rectangular base is:

$$p_1 = \frac{V}{A}\left(1 \pm \frac{6e}{d}\right) \qquad 4.1$$

where p_1 = contact pressure at upstream or downstream edge of base (in psf)

V = algebraic sum of all vertical loads and weights that act on the structure, including uplift (in lbs)

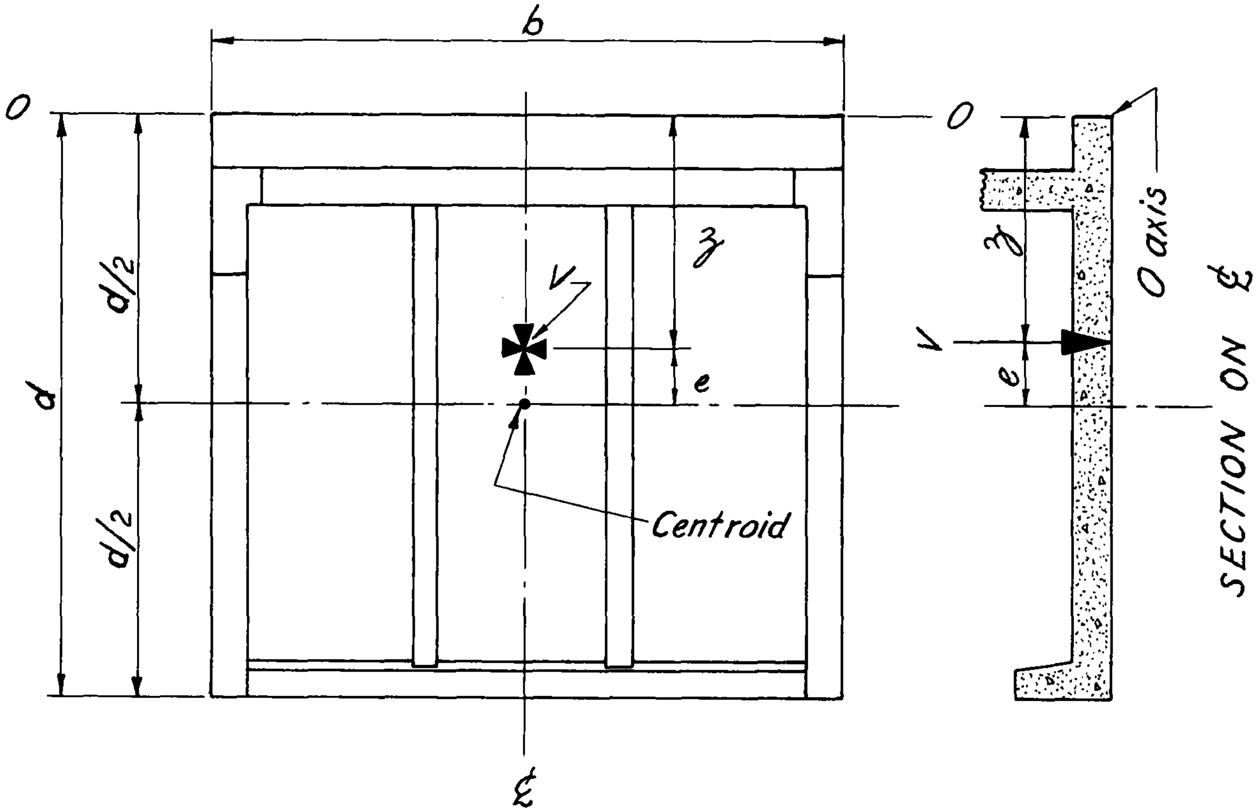
A = area of base on which contact pressures are assumed to act (in ft^2)

e = eccentricity = longitudinal distance between the centroid of the base area and the point of application of the resultant vertical load V (in ft)

d = base length = dimension from upstream edge to downstream edge of base area (in ft)

The centroid of the base area is on the longitudinal center line equidistance from the upstream and downstream edges of the base rectangle.

The area of the base, A, is equal to bd where b is the out-to-out transverse base dimension in feet.



PLAN OF BASE AREA

FIGURE 4.3

The total vertical load, V, includes the weight of all the concrete, earth above footings, water above any part of the structure under consideration, and uplift. Assume downward weights (loads) to be of positive sign; then uplift forces will be negative.

The location of the resultant V of all vertical forces including uplift can be found by taking moments about any arbitrarily selected axis. Select an axis 0-0 along the upstream edge of the base area at the elevation of the bottom of the apron. Let v_1 be the magnitude of a part of the vertical load or weight and ℓ_1 the perpendicular distance between its line of action and the 0-axis. Then the moment, M_v, of all such parts of the total vertical force, V, about the 0-axis is given by

$$M_v = v_1\ell_1 + v_2\ell_2 + \text{---}\ v_n\ell_n = \Sigma v_n\ell_n \qquad 4.2$$

and

$$V = v_1 + v_2 + \text{---}\ v_n = \Sigma v_n \qquad 4.3$$

Next compute the moment of all horizontal loads about the O-axis. Let h_1 be the magnitude of a part of the horizontal load and y_1 the vertical distance from its line of action through its centroid to the O-axis, etc. Then the moment M_h of all such parts of the total horizontal force H about the O-axis is given by

$$M_h = h_1 y_1 + h_2 y_2 + \text{---}\ h_n y_n = \Sigma h_n y_n \qquad 4.4$$

$$H = h_1 + h_2 + \text{---}\ h_n = \Sigma h_n \qquad 4.5$$

Then the distance z from the O-axis to the point of application of the resultant vertical force V is given by

$$z = \frac{M_h + M_v}{V} = \frac{\Sigma h_n y_n + \Sigma v_n \ell_n}{\Sigma v_n} \qquad 4.6$$

And the eccentricity e can be figured from relationships indicated in fig. 4.3 (page 4.13). The value of z may be either greater than or less than $(d \div 2)$. If $z > (d \div 2)$, the contact pressures at the toe or downstream edge of the base area will be greater than at the upstream edge.

The total resultant contact force acting on the foundation is made up of a vertical component V and a horizontal component H as determined by equations 4.3 (page 4.13) and 4.5 respectively. Obviously, the structure will float if the resultant V acts in an upward direction.

The uplift pressure diagram must be added algebraically to the contact pressure diagram, derived from equation 4.1 (page 4.12), to obtain the diagram of total pressures acting on the base.

The loading to be used in the design of the apron can then be determined by subtracting the weight of the apron and water above it from the total pressure diagram to give net apron load.

Piping. Piping may be defined as the removal of material from the foundation by the action of seepage water as it emerges from the soil below the dam. Failures by piping may result from subsurface erosion or heave. Subsurface erosion starts as a spring or springs near the downstream toe of the dam and progresses upstream along the base of the structure. Failure occurs when the upstream end of the eroded hole nears or reaches the upstream side of the dam. Failure by heave results when a large portion of soil near the downstream toe suddenly rises because the upward pressure of the seepage water is greater than the effective weight of the soil.

Unless the foundation is sealed with a watertight cutoff, water percolates through the foundation and emerges on the downstream side. The characteristics of the flow of this seepage water are similar to those of laminar pipe flow. The length of the path of flow and the frictional resistance to flow govern the outlet velocity of the seepage water and the pressures of the seepage water under the soil at the toe of the dam.

There are two schools of thought regarding the occurrence of seepage through earth foundations. One emphasizes the flow through the foundation material itself. The other believes that the line of least resistance is along the line of contact between the spillway and the foundation.

The "line of creep" theory produces the more usable method of design against failure by piping for structures of the size encountered in our work.

W. G. Bligh was one of the first engineers to advance the "line of creep" theory. It has been revised and refined by E. W. Lane (see "Security from Underseepage--Masonry Dams on Earth Foundations," Trans. of American Society of Civil Engineers, Vol. 100, p. 1235, 1935, and discussion in "Handbook of Applied Hydraulics," Calvin V. Davis, McGraw Hill Book Co.). This theory is based on the conclusion that the "line of creep," i.e., the line of contact between the dam and cutoffs with the foundation, will produce less resistance to percolation than another path through the foundation material. It is believed that the difficulty of securing an intimate contact between the dam and the foundation material, and the danger of unequal settlement which tends to destroy such contact, make this line of contact the one which will provide the least resistance to the flow of water.

After an intensive study of numerous existing dams on earth foundations, Lane was convinced that the majority of failures due to piping occurred along the line of creep. He also found that the majority of failures occurred to dams that had inadequate or no vertical cutoffs. These findings led him to recommend the use of a weighted creep line in which horizontal contacts with the foundation and slopes flatter than 45°, being less liable to have intimate contact, are assigned only one-third the resistance value of steeper contacts. In other words, the weighted creep line is the sum of all the steep contacts, plus one-third of all the contacts flatter than 45°, between the headwater and the tailwater along the contact surface of the dam and foundation. Should the distance between the bottoms of 2 cutoffs be less than one-half the weighted creep distance between, twice the distance between the cutoffs should be used instead of the actual line of creep between them.

Lane's recommended weighted creep ratios (C_w), the ratio of the weighted creep distance to head, are given for various foundation materials in the following table.

Material	C_w
Very fine sand and silt	8.5
Fine sand	7.0
Medium sand	6.0
Course sand	5.0
Fine gravel	4.0
Medium gravel	3.5
Course gravel including cobbles	3.0
Boulders with some cobbles and gravel	2.5
Soft clay	3.0
Medium clay	2.0
Hard clay	1.8
Very hard clay or hardpan	1.6

TABLE 4.2

Lane's theory resolves itself into the following equation:

$$C_w = \frac{\Sigma L_H + 3\Sigma L_V}{3H} \qquad 4.7$$

where C_w = weighted creep ratio

L_H = horizontal or flat contact distances

L_V = vertical or steep contact distances

H = head between headwater and tailwater

The foundation materials as listed in Lane's table do not coincide with the descriptions generally used in our work. Therefore, to obtain a working tool more applicable to our problem and to incorporate our past experience with erosion control structures, the following table of weighted creep ratios is recommended for our use.

Material	C_w
Clean gravel	5.0
Clean sand or sand and gravel mixture	6.5
Very fine sands and silts	8.5
Well-graded mixture of sand, silt, and less than 15 percent clay	5.5
Well-graded mixture of sand, silt, and more than 15 percent clay	4.0
Firm clay	2.3
Hard clay	1.8

TABLE 4.3

The appurtenances generally used in conjunction with drop spillways to guard against piping are the upstream blanket or fill, the upstream cutoff wall, and the downstream toewall. The upstream blanket should always be used in drop spillway construction when seepage is a problem, as it is an easy and economical means of protection. The upstream blanket also reduces the uplift pressures on the structure. The upstream cutoff wall serves three purposes--it safeguards against piping, reduces uplift pressures, and resists sliding. The downstream toewall serves two purposes--it safeguards against piping and protects the apron from undermining. The toewall has one detrimental effect--it increases the uplift pressures. Therefore, where deep cutoffs are required to safeguard against piping, it may be necessary to increase the depth of the upstream cutoff wall and decrease the depth of the downstream toewall to control the uplift pressures.

Mr. Streiff, in his discussion of Mr. Lane's paper, argues mathematically that weep holes have very little, and only localized effect in reducing pressures. Therefore, weep holes used in the sidewalls and headwall of drop spillways will be disregarded as far as piping and uplift are concerned.

So many indeterminate variables affect the design for safety against piping that a large factor of safety is mandatory. When one considers that the coefficient of permeability varies from about 10 cm per sec for coarse gravel to 10^{-9} cm per sec for dense clay, the complexity of the problem is apparent. Other factors that affect the problem are methods of construction and the variation of materials in the foundation. If various materials are encountered in the foundation, the material having the largest weighted creep ratio should be considered as the foundation material and the design made accordingly. It is almost presumptuous to point out that adequate foundation investigations are mandatory for safe dam design.

As pointed out previously, not all structure locations present a danger of piping. If the spillway is located on a deep, permeable foundation, with a low-water table, seepage from above the dam passes downward in a nearly vertical direction until it merges with the water table. Since there is very little, or no, tendency for this seepage to flow under the spillway and emerge in the downstream channel, the problems of uplift and piping do not exist. If soil borings and the geology of the site do not positively indicate the above conditions, the high-water table condition, discussed previously, should be assumed for design purposes.

Where the water table is high, or where the high-water table condition exists because of relatively impervious layers in the foundation near the apron elevation, the data contained in table 4.1 (page 4.5) and the previous discussion on uplift can be used in the solution of the piping problem. Procedure is illustrated in example 4.3.

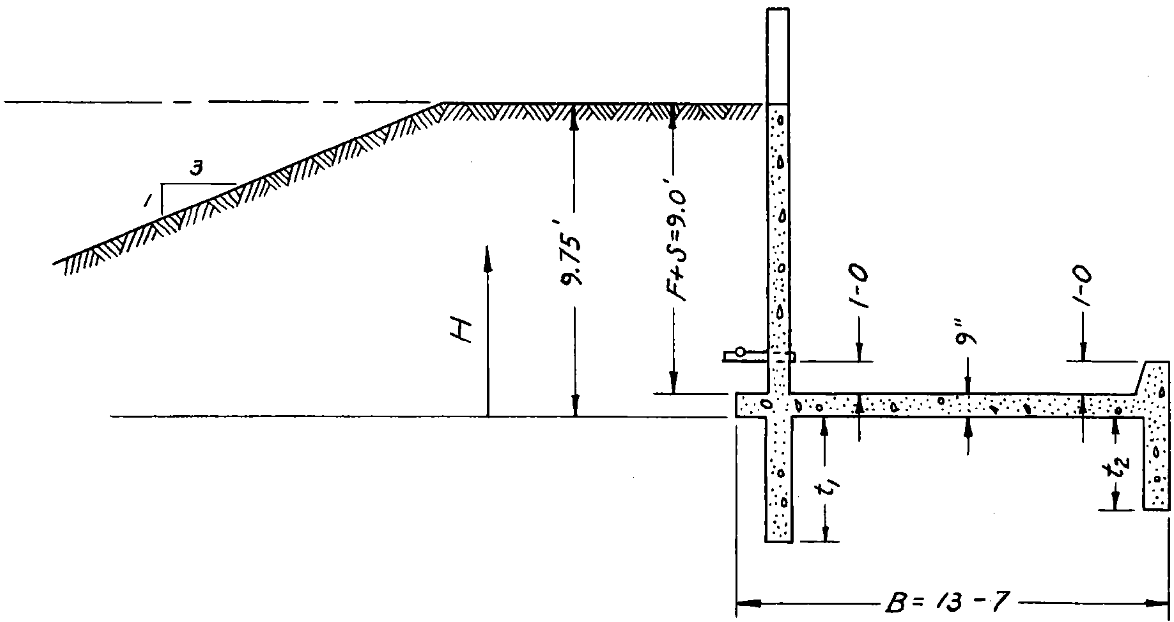
Cutoff walls may be constructed of reinforced concrete, interlocking steel sheet piling, pressure-treated Wakefield timber piling, or dense, well-compacted, impervious earth fill, or combinations of the above. The design of the cutoff will depend upon foundation conditions, availability of materials and construction equipment, cost, and other factors. Wakefield piling over 10 feet long is apt to cause trouble. Steel sheet piling works very well to considerable depth unless large rock and boulders are encountered. If the foundation is dry and the soils are stable at the time of construction, concrete cutoff walls up to 10 feet in depth should not cause undue trouble. If the foundation soils are saturated and the water table cannot be lowered, the construction of concrete cutoff walls to depths of 5 feet can be troublesome and costly. Earth cutoffs, to be impervious and effective, must be made of carefully selected, well-graded materials, placed at proper moisture content, and thoroughly compacted to high density; this is difficult to accomplish, especially if the foundation is wet. Hence, impervious earth cutoffs should normally not be used unless rock and boulders prevent the placement of driven piling; even then, it might be advisable to excavate the rock and boulders to required depth, then backfill with compacted earth and drive sheeting through it to obtain a reasonably watertight cutoff wall.

The cutoff wall must be securely connected to the remainder of the spillway and this connection must be watertight.

<u>Example 4.3</u>

Given: Drop spillway, $F = 8.0$ ft, $h = 3.0$ ft, $s = 1.0$ ft (see sketch, page 4.18). The foundation material is a well-graded mixture of sand, silt, and clay. Clay content, 20 percent. The relative permeability of the foundation and the fill material is estimated to be equal. A high-water table exists.

Find: The required depth of cutoff wall to insure against piping, if the depth of the toewall (t_2) is taken as 3.0 ft, for the following conditions: (1) Pond above structure with no upstream berm against headwall (2) Pond above structure with upstream berm and type (a) drainage, and (3) Pond above structure with upstream berm and type (b) drainage, fig. 4.1 (page 4.3).



Solution: $C_wH = B/3 + 2t_1 + 2t_2 = \frac{13.58}{3} + 2t_1 + (2 \cdot 3)$

$C_w = 4.0$ for foundation material

$\therefore = 2t_1 = 4H - 4.53 - 6.0$

$t_1 = 2H - 5.26$

1. No upstream berm

   From sketch H = 9.75

   $\therefore t_1 = 19.50 - 5.26 = 14.24$ ft

2. Upstream berm and type (a) drainage
From table 4.1 (page 4.5), Case C, no flow

   $y_2 = s + 0.5F = 1.0 + 4.0 = 5.0$ ft

   $H = 5.0 + 0.75 = 5.75$

   $\therefore t_1 = 11.50 - 5.26 = 6.24$ ft

3. Upstream berm and type (b) drainage
From table 4.1 (page 4.5), Case C, no flow

   $y_2 = s + 0.15F = 1.0 + 1.2 = 2.2$ ft

   $H = 2.2 + 0.75 = 2.95$

   $\therefore t_1 = 5.90 - 5.26 = 0.64$ ft

Use $t_1 = 2.5$ ft (minimum depth of cutoff wall)

This example indicates the effect of the earth berm and a good drain.

If the tailwater elevation is adequate to provide the desired energy dissipation in the stilling basin of the drop spillway, the maximum head tending to cause piping will occur when there is no flow over the structure.

The danger of piping due to horizontal percolation around the headwall extension must also be considered. If the relative permeability of the abutment material is equal to, or less than, the foundation material, the minimum length of the headwall extension should be 3 times the average depth of the cutoff wall and toewall below the bottom of the apron. If the relative permeability of the abutment material is greater than the foundation material, the minimum length of the headwall extension should be 3 times the average required depth of the cutoff wall and toewall, assuming that the foundation is made of the abutment material. In the second case, in lieu of extending the headwall extension, a core trench could be excavated into the abutment to the elevation of the bottom of the cutoff wall and backfilled with a material that is considerably more impervious than the foundation material. The core trench should extend into the abutment, measured from the end of the headwall extension, a distance equal to twice the length of the headwall extension. The minimum bottom width of the core trench should be 4.0 feet and the side slopes should not be steeper than one-half horizontal to one vertical.

Overturning. The structure is safe against overturning if positive contact pressures exist over the entire base area.

Uplift. As pointed out previously, the total weight of the structure plus all vertical downward forces acting on it must be greater than the uplift forces. Should the uplift be greater than the downward force, the structure will tend to float--a situation which, obviously, cannot be tolerated.

Sliding. The horizontal forces acting on the structure in the downstream direction have a tendency to slide the structure. The horizontal resisting forces must be sufficient to withstand this tendency with a margin of safety.

In the case of a drop spillway designed and constructed as a monolithic unit, the forces resisting sliding are the frictional resistance of the foundation, the friction resistance between the sidewalls and the earth fill, the passive resistance of the earth downstream from the toewall and headwall extensions, and, during times of flow, the hydrostatic pressure of the tailwater against the headwall and wingwalls.

Past field experience indicates that drop spillways, with F equal to 10 feet or less and with headwall extensions poured monolithically with the remainder of the spillway, are safe against failure by sliding if the minimum requirements of the depth of cutoff walls and length of headwall extensions are met.

In the design of large drop spillways, however, sliding must be considered and the design made with a liberal safety factor. It is possible to make reasonable estimates of the total possible resisting forces, but it is impossible to ascertain the distribution of the actual required forces to maintain the structure in equilibrium. The designer, therefore, does not know what the design loads should be for various parts of the structure. It is wise, therefore, in large structures to design the headwall extensions and wingwalls articulate from the rest of the structure, and provide watertight joints at the junctions.

The following procedure is recommended for computing stability against sliding. The plane of sliding is assumed to be on a plane between the bottom of the cutoff wall and the bottom of the toewall. The passive resistance of the earth downstream from the toewall is neglected. A safety factor of 1.5 is recommended. Therefore, the ratio of horizontal resisting forces to the total downstream forces should be equal to, or greater than, 1.5.

Figure 4.4 shows a cross section of the headwall and apron of a drop spillway and the forces that act on a longitudinal slice of the spillway. For any loading condition the horizontal force acting above the bottom of the cutoff wall in the downstream direction is H. The total downward vertical force, V, is the weight of the structure, minus uplift, plus the effective weight of the soil between the cutoff wall and the toewall above the plane of sliding. For equilibrium to exist, the vertical component of the resultant reaction, R_V, must equal V and the horizontal component, R_H, must equal H. The force resisting sliding, R_H, is made up of two parts, the friction force, fV, and the cohesion force, cA, so that

$$H = R_H = fV + cA \qquad 4.8$$

where R_H = horizontal resisting force in lbs

f = tan ϕ, the coefficient of friction

ϕ = angle of internal friction of foundation material

V = total vertical load in lbs

c = cohesion resistance of foundation material in lbs/ft^2

A = area of plane of sliding in ft^2

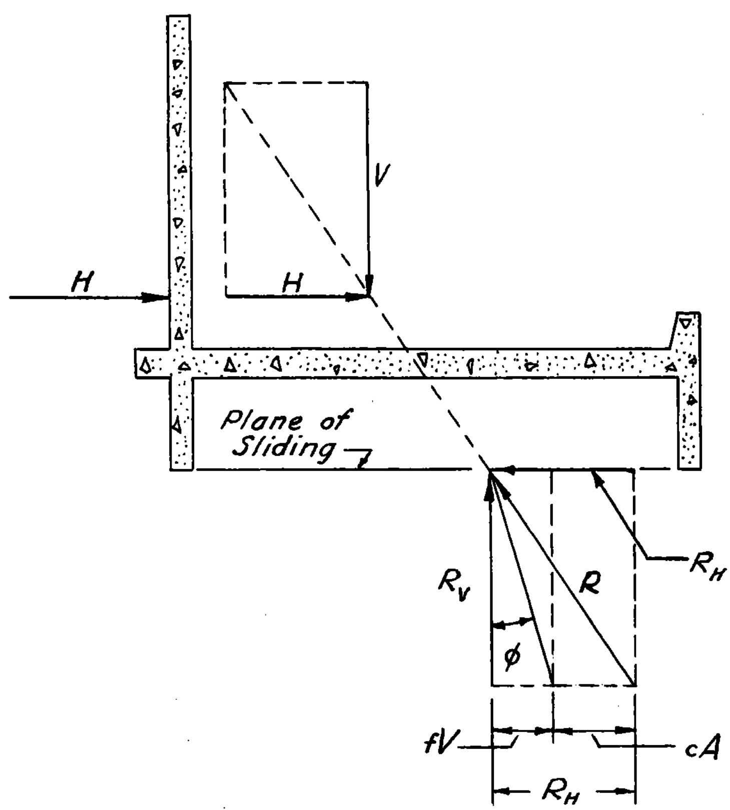


FIGURE 4.4

To provide a safety factor of 1.5, it is obvious that $fV + cA$ must equal 1.5H. If it is not possible to meet this criteria with a cutoff wall and toewall of reasonable depth, it will be necessary to provide an anchor whose pull or resistance to sliding, T, will satisfy the equation

$$T = 1.5H - fV - cA \qquad 4.9$$

If an anchor is provided, it must be placed on a level with the apron and upstream from the headwall a distance equal to or greater than that given by the formula

$$X = (F + s)\cot\left(45^\circ - \frac{\phi}{2}\right) = \frac{F + s}{\tan\left(45^\circ - \frac{\phi}{2}\right)} \qquad 4.10$$

where X = minimum distance from headwall to anchor in ft

ϕ = coefficient of internal friction of saturated backfill above the spillway

F + s = vertical distance from crest of spillway to top of apron in ft

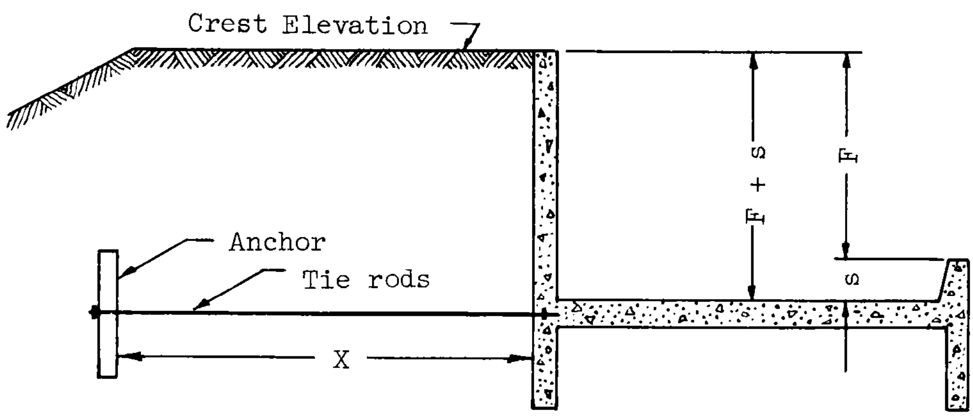


FIGURE 4.5

Passive pressures on the anchor can be computed from equations in paragraph 2.2.3, Engineering Handbook, Section 6 on Structural Design. In applying these equations, submerged weight of the backfill must be used for w.

Codes and Criteria. The design codes and criteria to be followed are given in the Engineering Handbook, Section 6 on Structural Design.

Headwall Analysis. The headwall may be designed as a slab considered fixed on three edges and free at the top in accordance with the Portland Cement Association Publication, "Rectangular Concrete Tanks," (ST-63). Drawing ES-6, Engineering Handbook, Section 6 on Structural Design is a plot of the moment and shear coefficients taken from this reference.

Sidewall Analysis. The procedure for design of the sidewall depends on the angle between it and the wingwall and on whether the sidewall and wingwall are monolithic or not. Three cases are cited below.

1. Monolithic with straight wingwall.

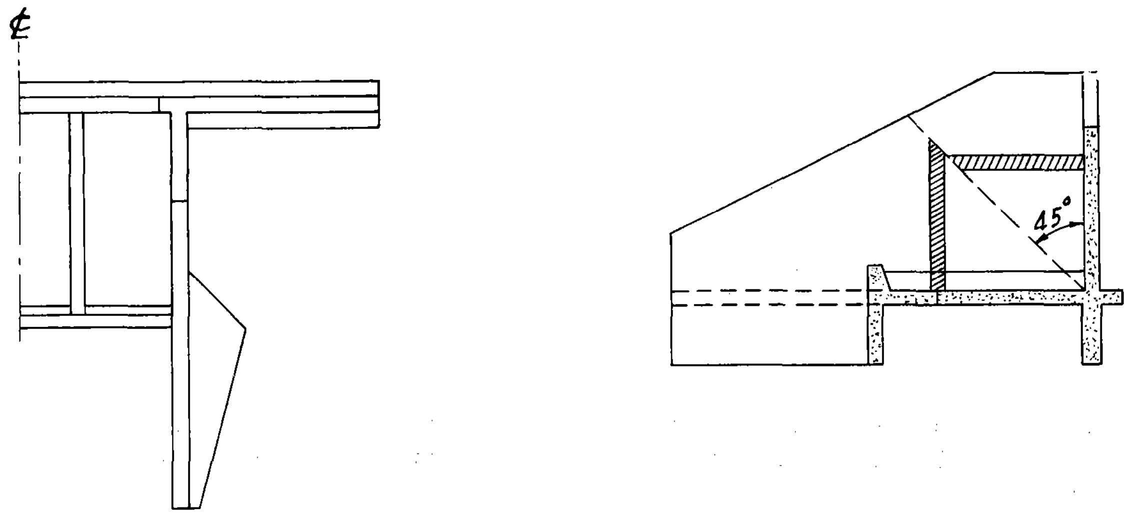


FIGURE 4.6

This sidewall may be assumed to act both as a horizontal and vertical cantilever, with the load between these two structural elements divided by a 45° line from the lower upstream corner of the wall as indicated in the sketch.

2. Sidewall and wingwall not monolithic, with angle between the two walls (β) between 0° and 45°.

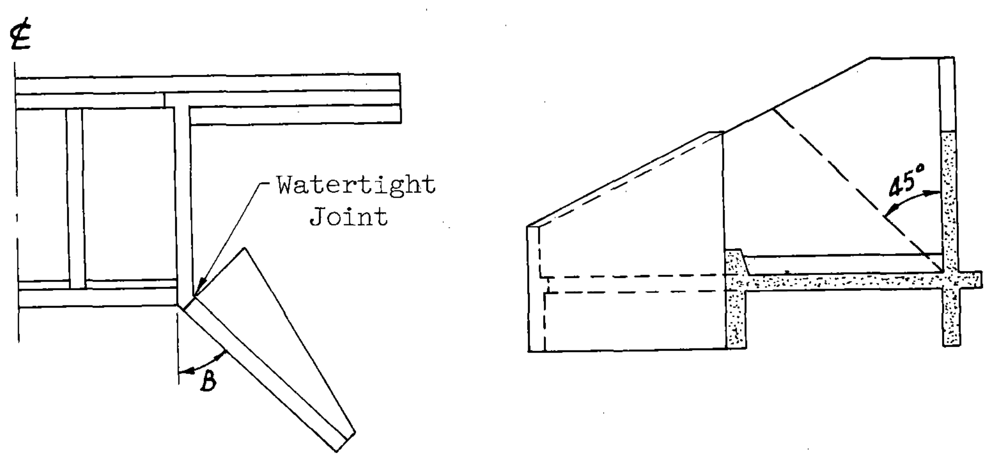


FIGURE 4.7

This sidewall may be designed in the same manner as the previous example.

3. Sidewall and wingwall monolithic, with 45° angle between the walls.

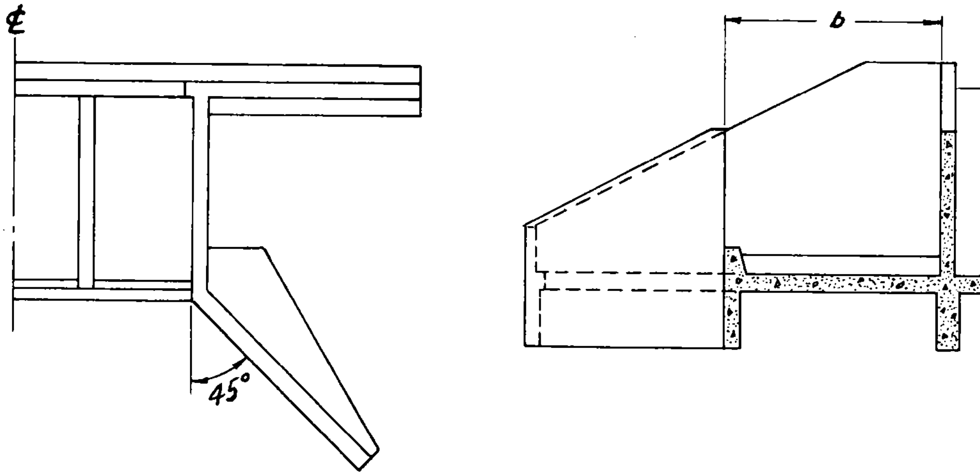


FIGURE 4.8

The following procedure provides an economical design, and results in the placement of reinforcing steel where past experience and good judgment indicate that it should be.

The basic assumptions for the design of the sidewall and wingwall are as follows:

(a) The sidewall is a slab fixed along its boundaries with the headwall and the apron. The downstream vertical edge is assumed to be supported and partially restrained by the wingwall. The top edge is free.

(b) The wingwall acts both as a vertical and horizontal cantilever, with the load between these two structural elements divided by the 45° line indicated in the sketch below.

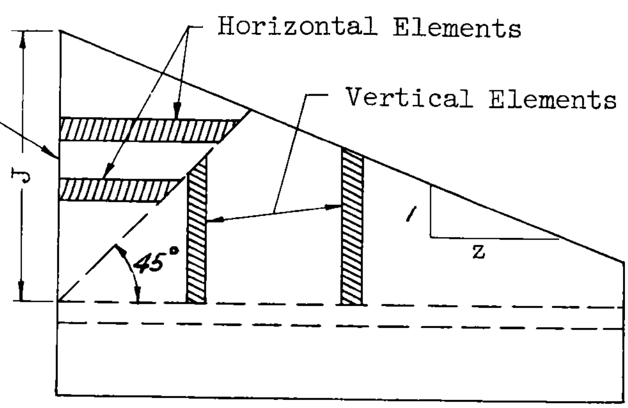


FIGURE 4.9

(c) The load distribution on both walls is triangular in the vertical plane normal to the wall.

The design procedure is outlined below by steps.

Step 1. Consider the vertical joint between the sidewall and wingwall as fixed against rotation and, from coefficients given in drawing ES-6, Engineering Handbook, Section 6 on Structural Design, determine the moments and shears in the sidewall for a slab fixed on three edges and free at the top. Take the average height of the sidewall as "a" for computing the b/a ratio.

Step 2. Compute maximum horizontal moment and maximum shear along the vertical edge of the sidewall in accordance with assumption (a), page 4.23.

$$M_s = C_m w a^3, \qquad V_s = C_s w a^2 \qquad 4.11$$

where M_s = maximum horizontal moment in ft lbs
V_s = maximum shear in lbs
C_m = moment coefficient from drawing ES-6
C_s = shear coefficient from drawing ES-6
w = equivalent fluid weight in lbs per ft^3
a = average height of sidewall in ft

Step 3. Compute maximum horizontal moment and maximum shear along the junction of sidewall and wingwall from the wingwall in accordance with assumption (b), page 4.23.

M_w is maximum when $y = J\left(\frac{z + 2}{3z + 2}\right)$

$$M_w = \frac{w(J - y)^2}{6z}\left[3zy - 2\,(J - y)\right] \qquad 4.12$$

V_w is maximum when $y = J\left(\frac{z + 1}{2z + 1}\right)$

$$V_w = w\left[y(J - y) - \frac{1}{2z}\,(J - y)^2\right] \qquad 4.13$$

where M_w = maximum horizontal moment in ft lbs
V_w = maximum shear in lbs
w = equivalent fluid weight in lbs per ft^3
J = height of wall from top of apron at junction of sidewall and wingwall in ft
y = distance from top of wall in ft
z = ratio of horizontal to vertical of the slope of the top of the wingwall

The above equations are derived from the load distribution assumptions previously described.

Step 4. Assume that the maximum moments and shears from the sidewall and and the wingwall occur on the same horizontal slice, and find the centroidal tension thrust in the wingwall and sidewall necessary to counteract the shearing fources from the wingwall and sidewall for the assumed one-foot horizontal slice.

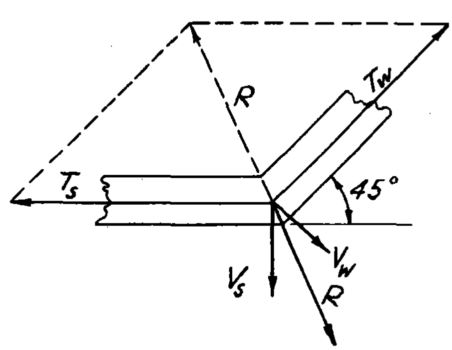


FIGURE 4.10

$$T_w = 1.414\ V_s + V_w \qquad 4.14$$

$$T_s = V_s + 1.414\ V_w \qquad 4.15$$

T_w = centroidal tension thrust in wingwall in lbs

T_s = centroidal tension thrust in sidewall in lbs

The following free body diagram shows all of the external forces acting on the joint of the above hypothetical slice.

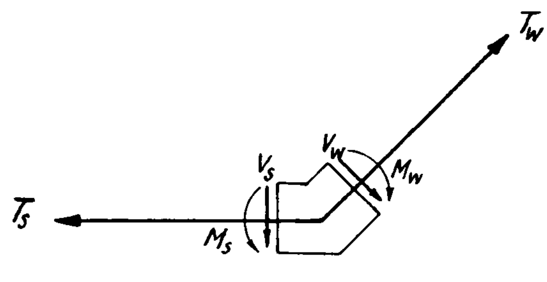


FIGURE 4.11

This joint is in equilibrium except for the moments which are unbalanced.

It is reasonable to assume that this and the other unbalanced moments along the junction of the sidewall and the wingwall will be distributed mainly into the apron, because of the rigidity of this general stress path. With this assumption, the release of the downstream vertical joint of the sidewall would have no effect on the moments along its other vertical joint at the headwall. This line of reasoning leads to the assumption that the maximum horizontal moment at the center of the sidewall should be increased by 0.5 $(M_s - M_w)$ as indicated in the sketch below.

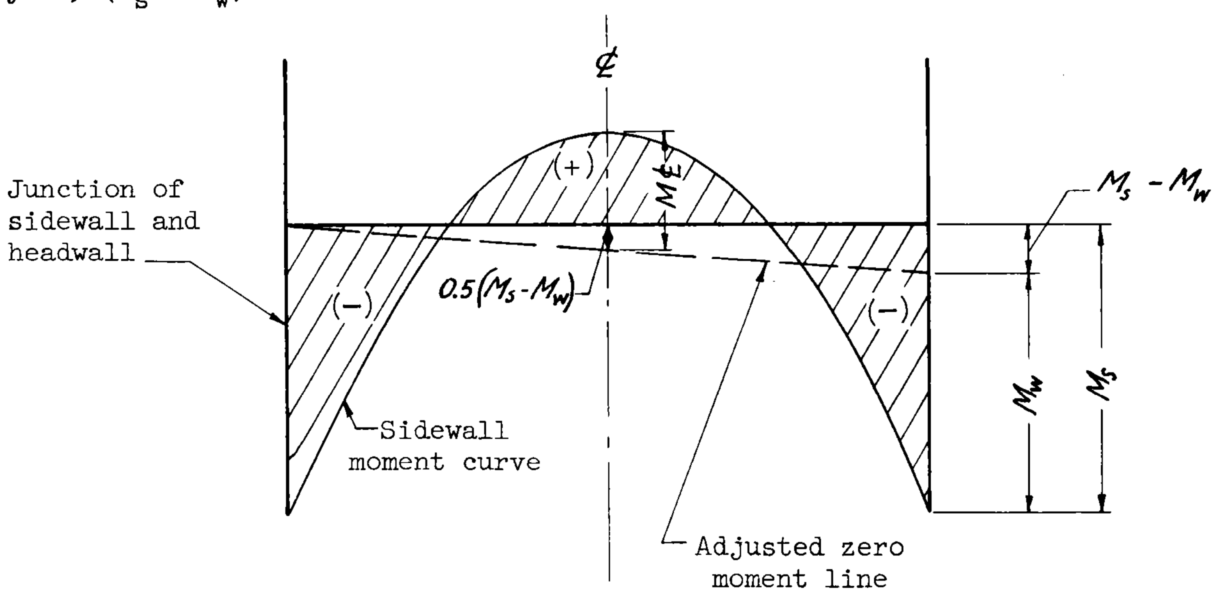


FIGURE 4.12

For conservative design, the sidewall and wingwall should be designed for the maximum moments and shears found from the above assumptions, whether they occur before or after the adjustment.

The horizontal steel in the exposed face of the sidewall will be designed for a moment, $M = C_m wa^3 + 0.5\ (M_s - M_w)$ for the full height of the wall.

The horizontal steel in the unexposed face of the sidewall at the upstream end of the wall will be designed for the maximum moment and shear as determined from drawing ES-6, Engineering Handbook, Section 6.

The horizontal steel in the unexposed face of the sidewall at the downstream end of the wall (junction with wingwall) will be designed to take the moment, M_S, plus the axial tension force, T_S. The steel required for M_S may be cut at the quarter point of the span, but the steel required by T_S should be extended to lap with the steel at the other end of the wall.

The principal vertical steel in both faces of the sidewall will be designed for the vertical moments determined from drawing ES-6.

The horizontal cantilever steel in the wingwall will be designed for the moment, M_S, plus the axial tension force, T_W.

The vertical steel in the wingwall will be determined from cantilever moments.

Wingwall Analysis. Refer to the three cases cited under sidewall analysis. The wingwall in the first two cases may be designed as a vertical cantilever. The wingwall analysis for case (3) is explained along with the sidewall design for this case. The required wingwall footing for all three cases is determined by considering the wingwall as an independent wall and making it stable against overturning.

In some cases it may be impracticable to provide sufficient resistance to sliding of the wingwall by frictional resistance on the bottom of the footing. Passive resistance on the toewall extension under the wingwall should be neglected because the fill in front of the toewall is apt to be wet and of low shearing strength when maximum loads are against the wingwall and because this fill may scour and be washed away. Where frictional resistance to sliding is not adequate the toewall, footings, and an upstream extension of the footing should all be poured monolithically with the apron and its toewall. Then the wingwall footing can be designed as a horizontal cantilever to transfer a part of the horizontal loads on the wingwall into the apron slab. This design procedure is illustrated in the structural design example.

Apron Analysis. The apron may be designed as a series of beams perpendicular to the sidewalls. The beams are considered as supported at the sidewalls and continuous with them, and continuous over the longitudinal sills or buttresses. Refer to pages 4.9 to 4.14 for the method of determining the apron loading. See drawing ES-56 (page 4.27) for moment and shear determinations.

Buttress Analysis. Buttresses for the headwalls of drop spillways of average size can usually be designed as cantilever beams. They should have a minimum width of 12 inches and a depth sufficient to carry the overturning moment which results from shears from the headwall. Vertical compressive stresses computed on the assumption of a rectangular beam should be corrected to give the maximum compressive stress parallel to the downstream face of the buttress. This method of analysis is illustrated in the structural design example.

The load to be used for the design of the buttress is the sum of the shears along the fixed vertical edges of the adjacent headwall slabs. For values of (b ÷ a) equal to or less than 2, the distribution and magnitude

DROP SPILLWAY APRON DESIGN: MOMENTS AND SHEARS

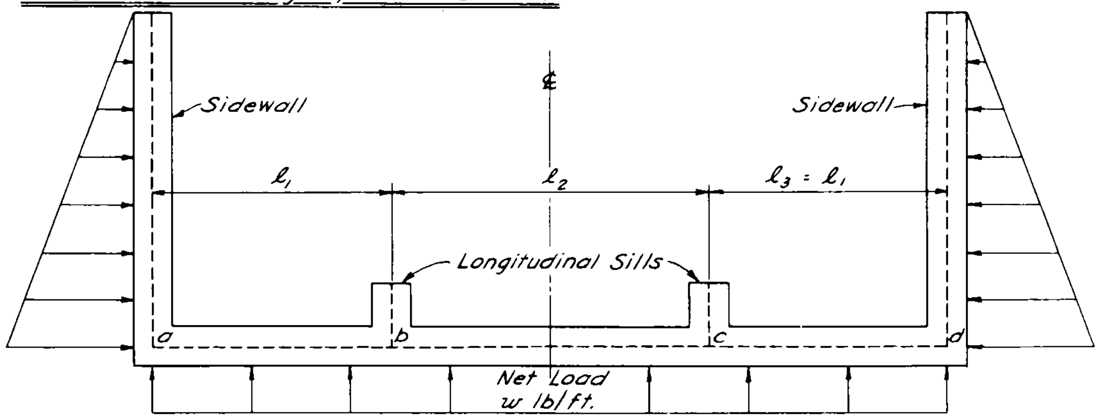
Assumptions:

Loads and structure are symmetrical about ℄ of structure. The analysis is based on ℄ dimensions of members.

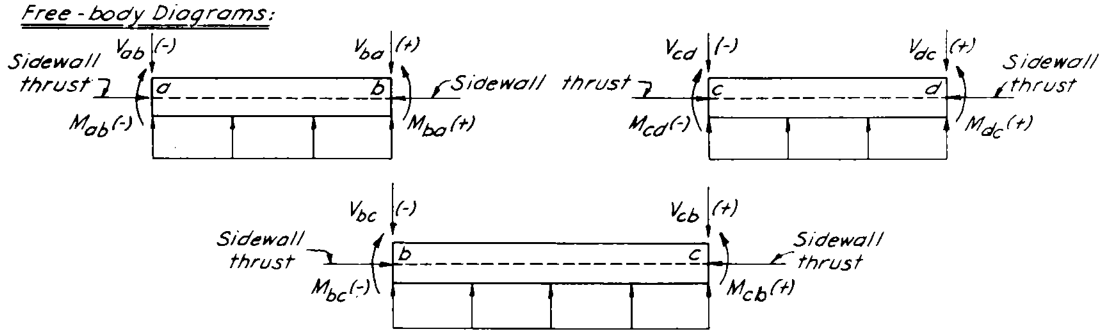
Sign Convention:

A moment acting in a clockwise direction on a joint is positive. Shear acting down on the left side of a section is positive.

Transverse Slice through Apron and Sidewall



Free-body Diagrams:



NOTE: M_{ab} and M_{dc} are equal to the corresponding vertical sidewall moments but are of opposite sign.

Moment Equations:

$-M_{ab} = M_{dc}$, $M_{ba} = -M_{bc}$, $M_{cb} = -M_{cd}$

When $\ell_1 = \ell_2 = \ell_3$

$$M_{ba} = \tfrac{1}{5} M_{ab} + \tfrac{1}{10} w\ell_1^2$$

When $\ell_1 = \ell_3$ and ℓ_2 is less or greater

$$M_{ba} = \frac{M_{ab}\ell_1 + \frac{1}{4} w(\ell_1^3 + \ell_2^3)}{2\ell_1 + 3\ell_2}$$

Shear Equations:

$$V_{ab} = -\tfrac{1}{2} w\ell_1 + \left(\frac{M_{ab} + M_{ba}}{\ell_1}\right) = -V_{dc}$$

$$V_{ba} = w\ell_1 + V_{ab} = -V_{cd}$$

$$V_{bc} = -\tfrac{1}{2} w\ell_2 = -V_{cb}$$

Caution: Use proper sign of moments and shears when substituting them in the given equations.

Example:

Given - $\ell_1 = 6.0$ ft, $\ell_2 = 8.0$ ft, $\ell_3 = 6.0$ ft, Vertical sidewall moment = 1600 ft. lb.

Problem - Find moments and shears by equations. Check moments by moment distribution procedure. Plot moment and shear diagrams.

Solution by equations

$M_{ab} = -1600$ ft. lb.

$$M_{ba} = \frac{(-1600)(6) + \frac{1}{4}(300)(6^3 + 8^3)}{(2)(6) + (3)(8)}$$

$$M_{ba} = \frac{-9600 + 54{,}600}{36} = +1250 \text{ ft. lb.}$$

$M_{bc} = -M_{ba} = -1250$ ft. lb.

$$V_{ab} = -\tfrac{1}{2}(300)(6) + \left(\frac{-1600 + 1250}{6}\right)$$

$V_{ab} = -900 - 58 = -958$ lb.

$V_{ba} = (300)(6) - 958 = 1800 - 958 = 842$ lb.

$V_{bc} = -\tfrac{1}{2}(300)(8) = -1200$ lb.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-56 SHEET 1 OF 2 DATE 8-6-51
Revised 3-11-53		

DROP SPILLWAY APRON DESIGN: MOMENTS AND SHEARS

Checking Moments by Moment Distribution

References: Theory of Simple Structures, Second Edition, by Shedd & Vawter.
No. ST40, Moment Distribution applied to Continuous Concrete Structures by Portland Cement Association.

3/4 Rule

	a	b	b	¢
		0.75x0.572=0.428	0.428	
+1600	-900	+900	-1600	
	-700 →	-350		
		+525	+525	
			-263	
		+132	+132	
			-66	
		+33	+33	
			-17	
		+8	+8	
			-4	
		+2	+2	
+1600	-1600	+1250	-1250	

Stiffness of members, $K = 4EI \div \ell$
$4EI$ is equal for all members
Let $4EI = 8$ then
$K_1 = 8 \div 6 = 1.333,\ K_2 = 8 \div 8 = 1.0$

Relative Stiffness

$$\text{of } \ell_1 = \frac{K_1}{K_1+K_2} = \frac{1.333}{1.333+1.0} = 0.572$$

$$\text{of } \ell_2 = \frac{K_2}{K_1+K_2} = \frac{1.0}{1.333+1.0} = 0.428$$

Fixed End Moments $= \frac{1}{12} w\ell^2$

$$M^F_{ab} = -M^F_{ba} = -\tfrac{1}{12}(300)(6)^2 = -900 \text{ ft. lb.}$$

$$M^F_{bc} = -M^F_{cb} = -\tfrac{1}{12}(300)(8)^2 = -1600 \text{ ft. lb.}$$

Plotting Moment and Shear Diagrams

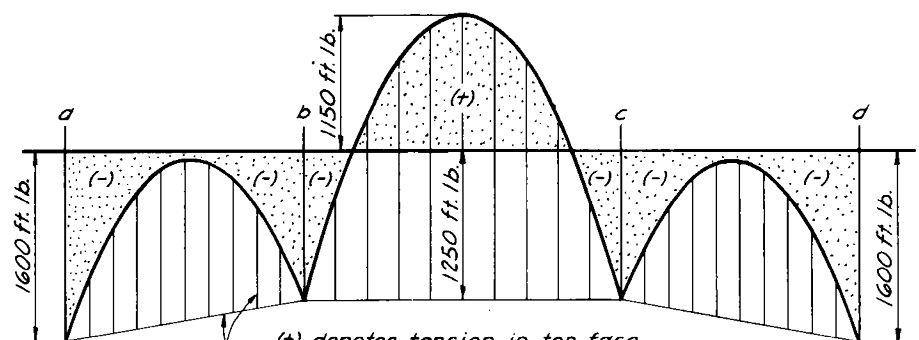
Simple bending moment $= \frac{1}{8} w\ell^2$

End span, $M^S = \frac{1}{8} w\ell_1^2$
$= \frac{1}{8}(300)(6)^2 = 1350$ ft. lb.

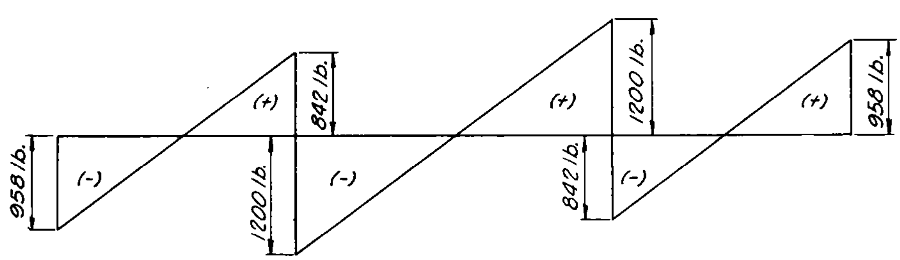
Center span, $M^S = \frac{1}{8} w\ell_2^2$
$= \frac{1}{8}(300)(8)^2 = 2400$ ft. lb.

x/ℓ = k	M_x / M@¢	Simple bending moments: In end spans	Simple bending moments: In center spans
0.1	0.36	486	864
0.2	0.64	864	1,536
0.3	0.84	1,134	2,016
0.4	0.96	1,296	2,304
0.5	1.00	1,350	2,400

See ES-1 in Structural Design Section (Section 6)



Moment Diagram



Shear Diagram

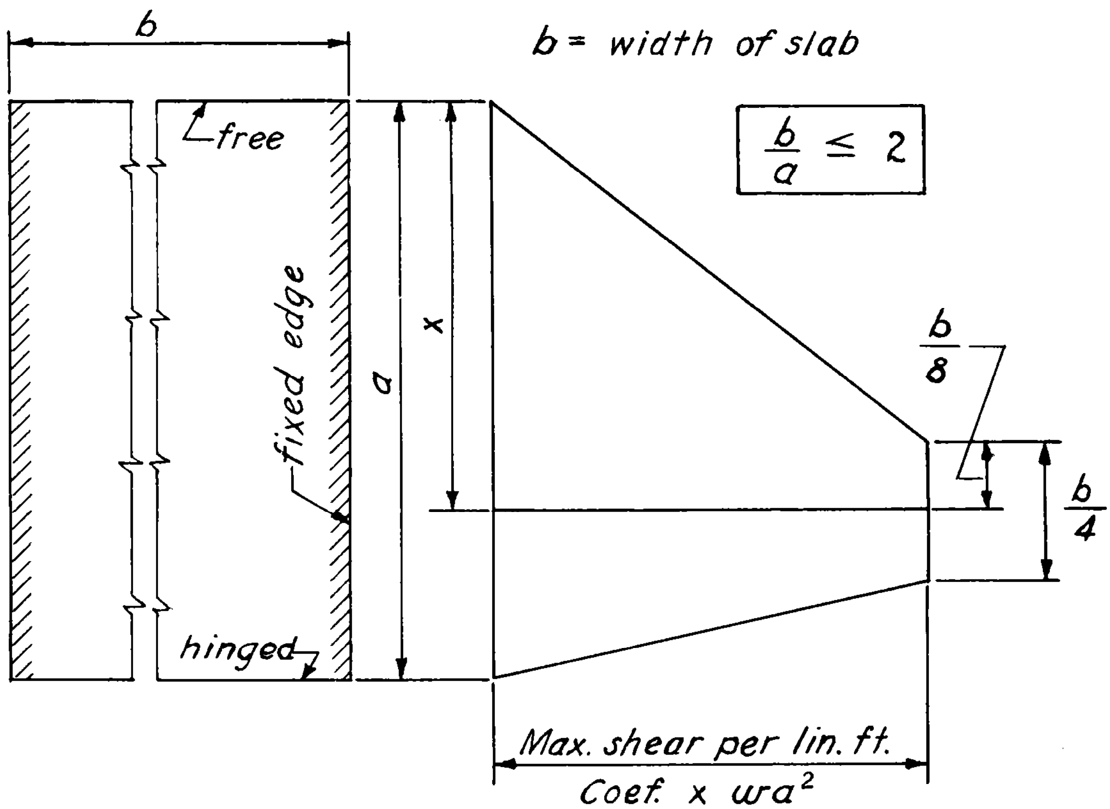
REFERENCE

Revised 11-8-51

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
H. H. Bennett, Chief
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-56
SHEET 2 OF 2
DATE 8-6-51

of the shear along the vertical edge of a headwall slab are given with reasonable accuracy by the following shear diagram, fig. 4.13.



DISTRIBUTION OF UNIT SHEARING STRESS ALONG FIXED EDGE

FIGURE 4.13

In fig. 4.13, the distribution of unit shearing stress is represented by a trapezoid, the area of which represents the total shear along the edge of the slab. The coefficient used to determine the maximum shear per linear foot and the value of x, which determines the location of the center of maximum shearing-stress intensity, are found from drawing ES-6, sheet 9 of 10, Engineering Handbook, Section 6 on Structural Design. The shearing-stress distribution shown in fig. 4.13 is good only for values of (b ÷ a) equal to, or less than, 2.

The total load is equal to the sum of the shears from the two adjacent spans. If the spans adjacent to the buttress are equal (the usual case), then the total load on the buttress is equal to twice the load indicated by the diagram in fig. 4.13.

When the overturning moments on the drop spillway are high and result in high toe pressures under the spillway, or when the weir length is relatively great, it may be necessary to devise special methods of analysis for the buttress, longitudinal sill, and transverse sill. Experienced structural designers should be consulted in such cases.

Longitudinal Sill Analysis. Longitudinal sills may be used with or without buttresses. In either case, the procedure of analysis is the same.

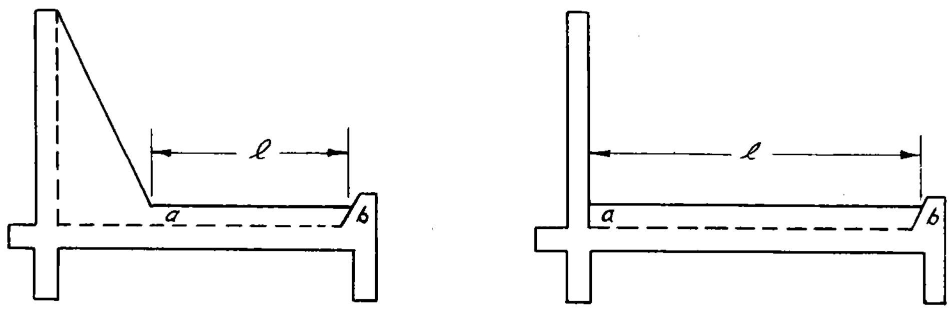


FIGURE 4.14

The longitudinal sill may be considered as a beam fixed against rotation at the toe of the buttress or at the headwall, point a, fig. 4.14, as the case may be, and as both partially restrained and freely supported at the transverse sill, point b, fig. 4.14. For the partially restrained condition at point b, the moment at b is taken as one-half the fixed end moment. The load on the longitudinal sill is taken as the maximum net reaction from the apron slab at the longitudinal sill less the weight of the sill.

Transverse Sill Analysis. When the longitudinal sills are designed as outlined in the preceding paragraph, the transverse sill acts as a support for the longitudinal sills. The analysis following these assumptions may be handled as follows: The transverse sill and toewall may be considered as a beam supported at the sidewalls. This beam should be designed for both fixed and half-fixed end moments; the actual degree of restraint at the end is unknown. The loads on the beam will be the reactions from the longitudinal sills as concentrated loads, plus a uniform load equal to the difference between the overturning pressure acting on the toewall and the weight of the beam.

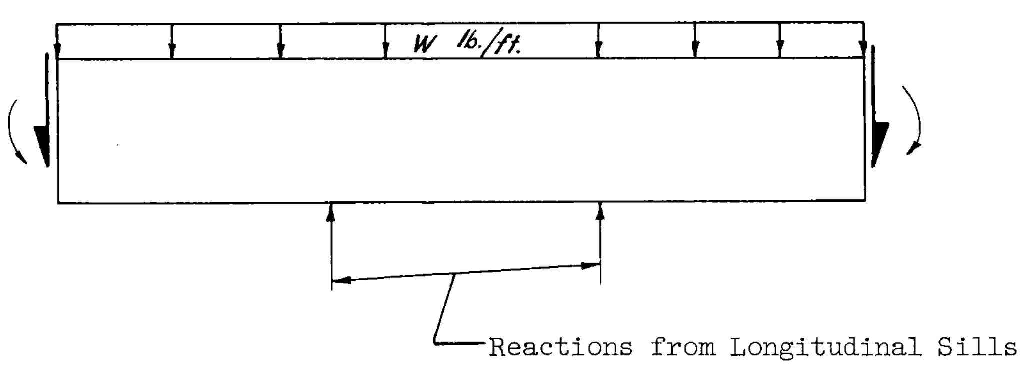


FIGURE 4.15

The resultant uniform load may act either in an upward or downward direction.

Headwall Extension Analysis. If the headwall extension is not joined monolithically with the rest of the structure, it acts merely as a diaphragm. The differential in earth loads on the two sides of the wall at any point will be very small. Therefore, the stresses in the wall will be small and the wall need only be reinforced to meet minimum steel requirements. This type of design for the headwall extension is used in the design example.

If the headwall extension is designed to be monolithic with the rest of the structure, it may be designed as a vertical and horizontal cantilever. See page 4.9 for load recommendations. In the vertical direction, the wall may be designed as a series of cantilever beams. In the horizontal direction, the wall and footings will be designed as a unit. The steel in the footings will be designed to carry the remainder of the total moment not carried by the horizontal steel in the wall.